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TECHNICAL MEMORANDUM 301

DIGITAL COMPUTING FACILITY OF U. S. NAVAL ORDNANCE TEST STATION

by

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INTRODUCTION

Experience in operating a technical computing facility consisting of IBM (International Business Machines) punch card equipment has disclosed that many scientific personnel who have occasional need for large scale computing services have no clear concept of either the computational power and speed or the limitations of such a facility. This report has been prepared to disseminate to present and potential users of the services available in the IBM computing laboratory at the Naval Ordnance Test Station understandable information on the characteristics of the various items of equipment in that laboratory and on the kinds of problems which can be handled. It is hoped that with such an understanding more people will be able to evaluate realistically whether their problems can advantageously be approached with numerical methods which utilize the speed, capacity, and flexibility of the available equipment.

The first section of the report introduces the punch card and the standard accounting type IBM machines which use the punch card. These machines include the Key Punch, Sorter, Collator, Reproducer, and Tabulator. Section two is devoted to the Calculating Punch, following its development from the electro-mechanical 601 to an improved model 602, and finally to the latest type 604 Electronic Calculating Punch. Section three considers several problems which could be handled on the equipment described in the first two sections. In the final section

there are descriptions of the standard and improved Card Programmed Calculator (CPC) models, now in local use, which are the most advanced IBM computing units available. Applications of these digital computers to various types of computational problems are given.

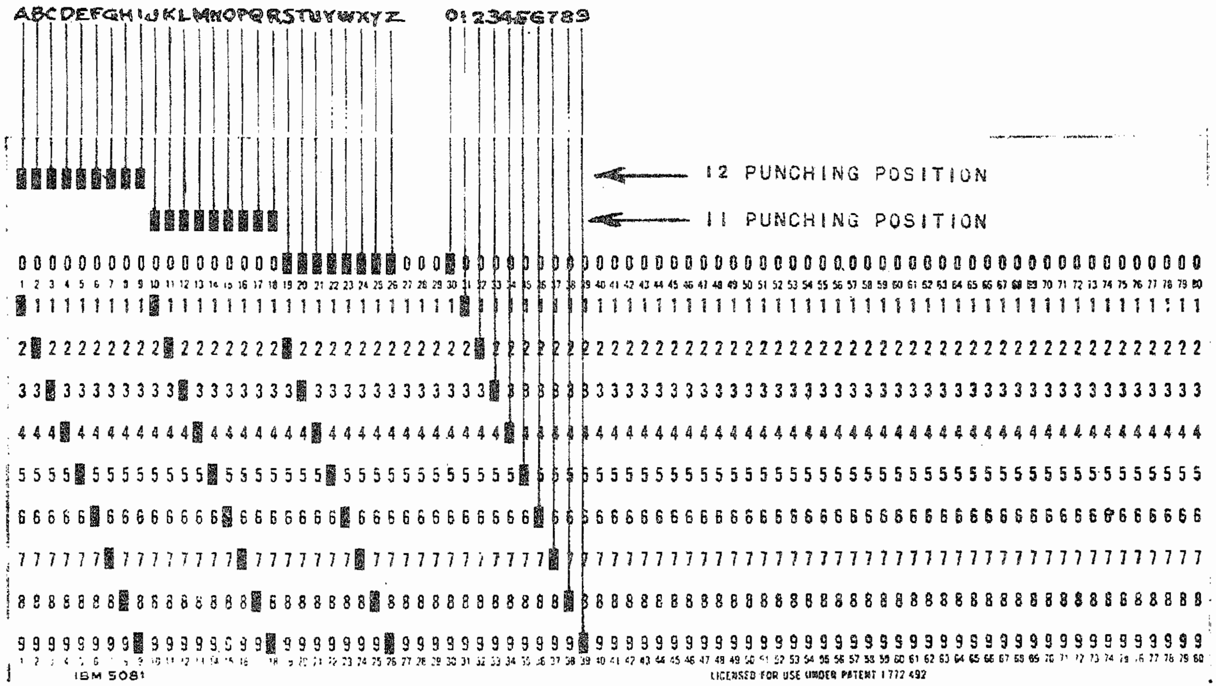
STANDARD PUNCH CARD EQUIPMENT

The IBM Card.

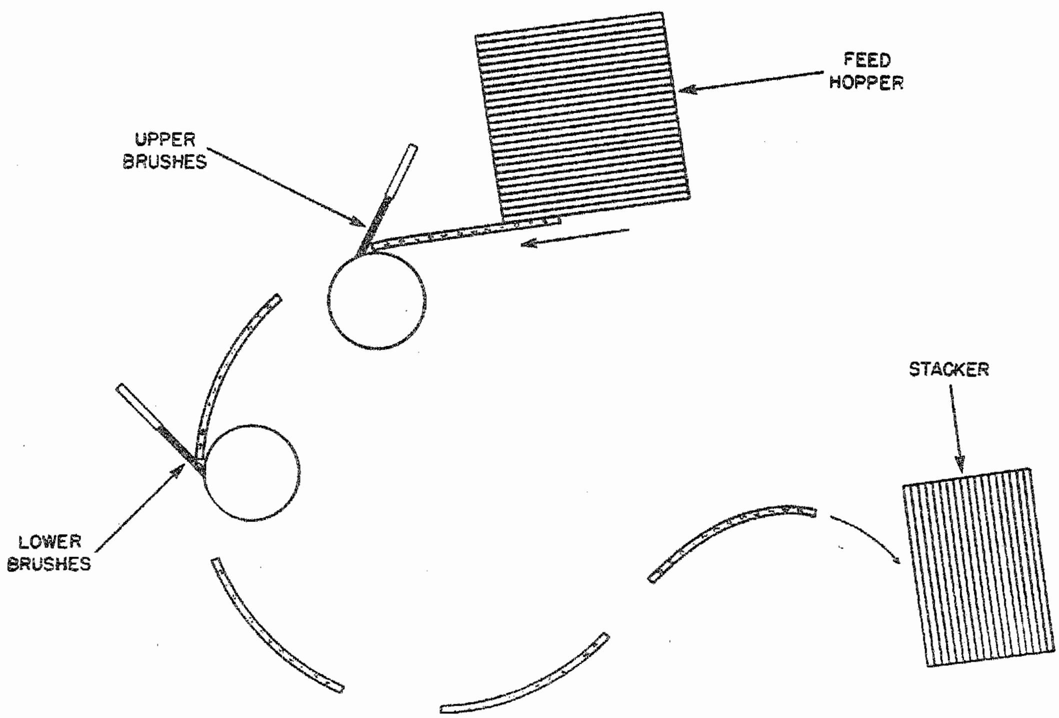
The International Business Machines Corporation uses the type of card shown in Fig. 1 in all of its modern punch card machines. Fig. 1 shows a standard 80-column card in which either digits or letters can be punched. The letters of the alphabet are shown punched in columns 1-26, while columns 30-39 show the digits 0-9 punched on the card. These cards are fed through the various machines. Each machine reads a digit or a number of digits that are punched in the card, does a particular operation, and then stacks the cards in order ready to be used in a different machine. For example, Fig. 2 shows the card feed of an IBM Tabulating Machine. The cards pass from the feed hopper, over the rollers, underneath the two sets of brushes and into the card stacker. Electrical contact with the roller is made when a brush in a particular column passes over a hole in the corresponding column of the card. The timing of this electrical impulse is used to distinguish the various digits and letters.

Key Punch.

Original data, constants, identification numbers, and machine instruction coding are punched on the cards through the use of the Key

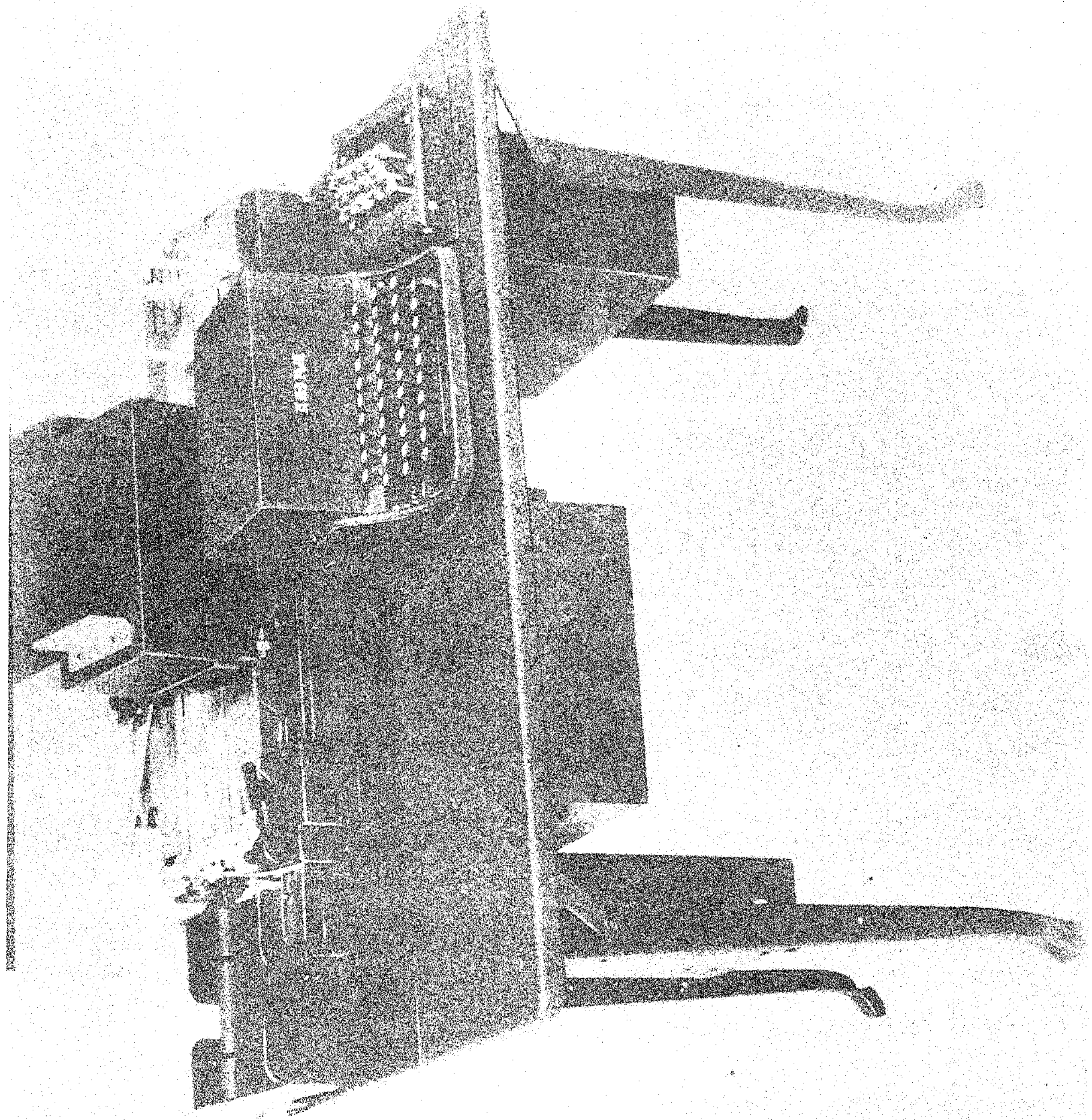


I.B.M. CARD
FIG. 1



Schematic Diagram of Card Feed

Fig. 2



KEY PUNCH
FIG. 3

Punch shown in Fig. 3. The cards feed automatically from the right-hand card hopper of the machine, pass through the punching mechanism, and are stacked in the left-hand card stacker. The large keyboard with a standard typewriter layout may be used to punch letters and numbers; the small keyboard may be used to punch numbers only. Cards will feed automatically, stopping in any column predetermined by the operator, who then key-punches the desired numbers or letters into these columns. When the operator has finished punching a card, it will pass to the card stacker at the same time that a new card is entering the punch. The Key Punch will duplicate a card automatically, and is used for this purpose when a small number of cards are to be made over. After the cards are punched, they are ready to be used in other IBM machines, each of which is built to perform a specialized operation.

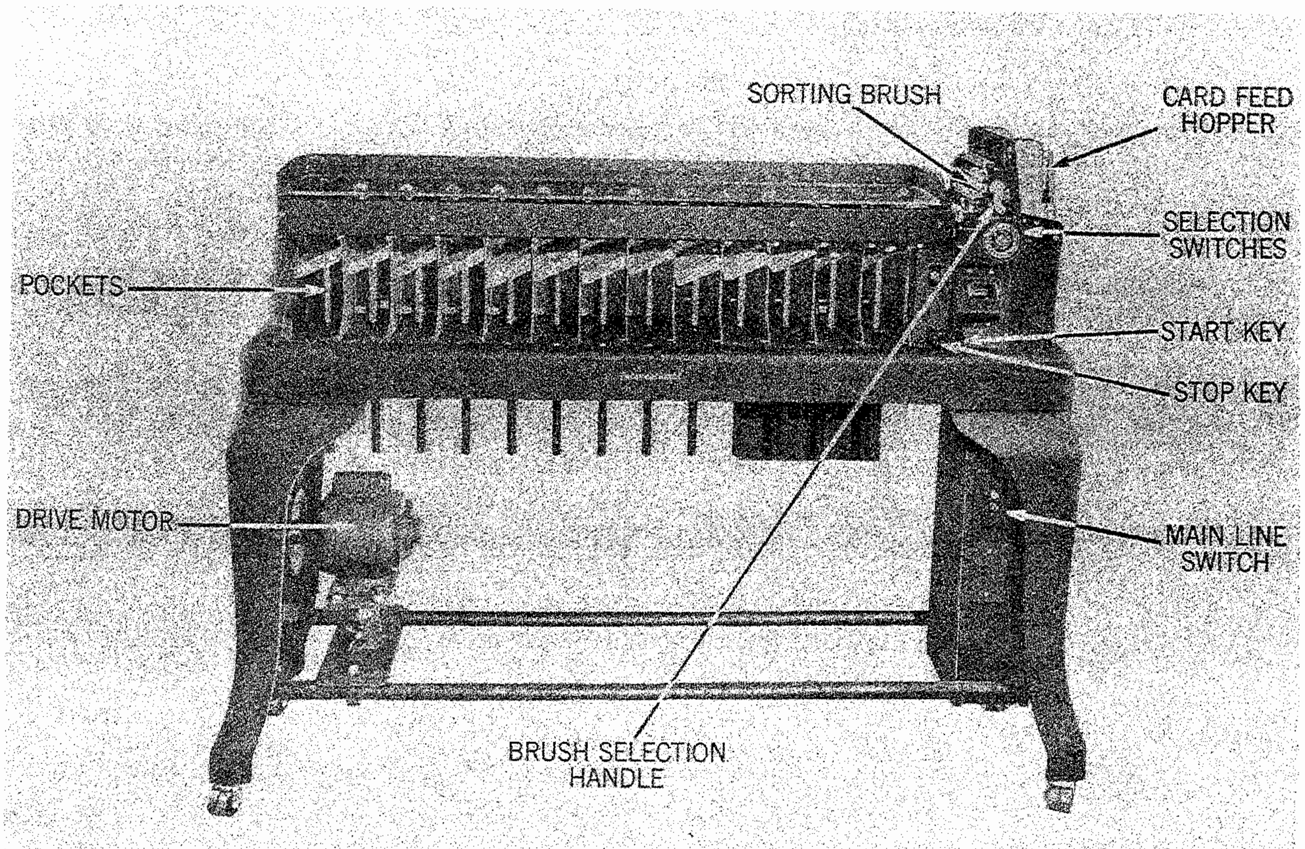
<u>Sorter</u>.

The Sorter is shown in Fig. 4 doing a sorting operation. The cards to be sorted can be seen in the right-hand card feed hopper and the cards that have already passed through the sorter are in the various pockets. Fig. 5 illustrates the operating principle of the Sorter. In the upper part of this figure, the reading brush is shown (A) reading a punch card whose front edge is at the point E. The lower part of the figure shows the reading brush making contact with the metal roller through a hole in the card. This activates the magnets (D) which open the metal chute allowing the card to enter the opening

between the number four and the number five chute blades. The digit that is punched in the cards thus determines which pocket the card will enter. The Sorter is used primarily for putting groups of cards into a specified order, or selecting particular cards from a large group. The Sorter senses on a single column at a time, and operates at a speed of 450 cards per minute. An improved IBM Sorter has recently been put on the market which operates at a speed of 650 cards per minute. (The Sorter will feed more cards per minute than any other IBM machine.) A large group of cards can be speedily sorted by running the cards through several times, sensing a different column on each run. For example, a group of 900 cards could be put in numerical order by running them through the Sorter three times, sorting first on the units column, then on the tens column, and finally on the hundreds column.

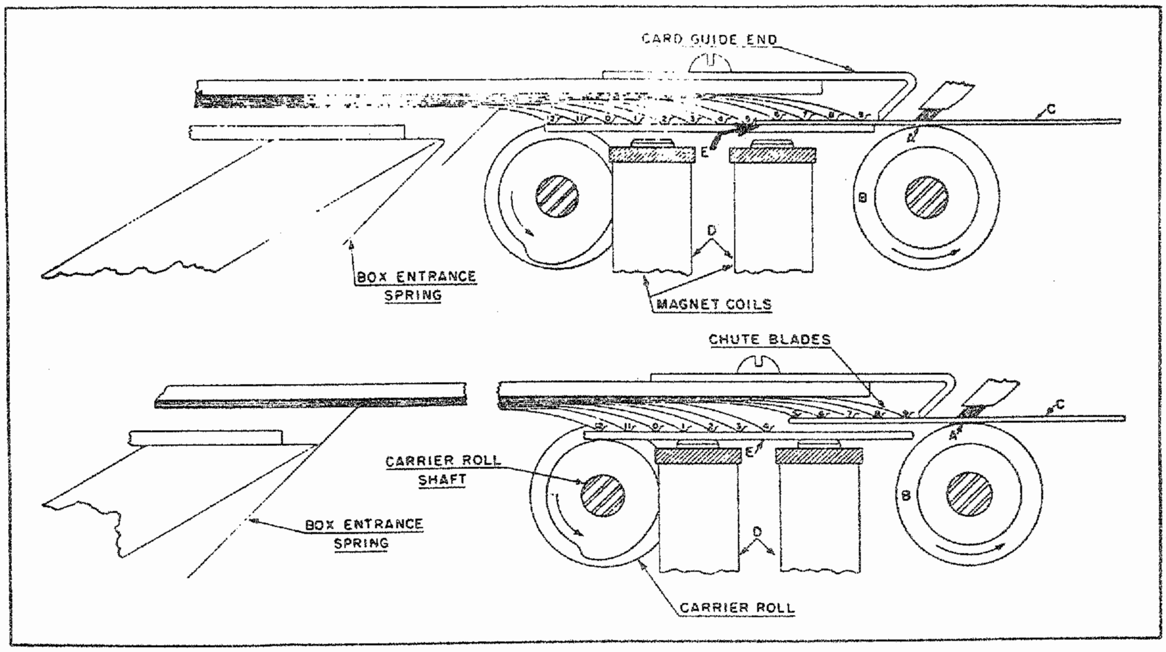
Collator.

The Collator is used primarily for merging, matching, checking and counting IBM cards. The operating principle of the Collator is illustrated in Fig. 6. The cards feeding from the primary and secondary hoppers are read by the three sets of brushes. The sequence and selector control units then determine in which of the four pockets a particular card will be placed. Fig. 7 shows the collator during a selective operation. Cards are shown entering from the primary and secondary card feed hoppers, while the cards that have passed through the Collator are shown stacked in the four left-hand card pockets.



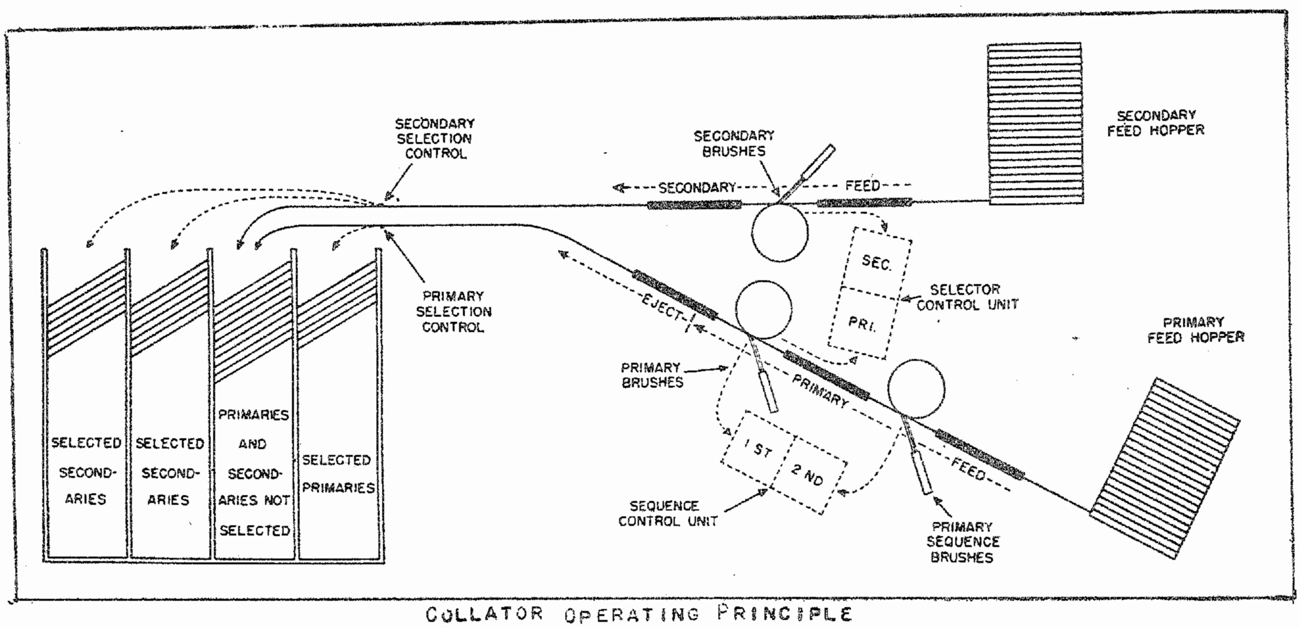
CARD SORTING MACHINE
Type 080

Fig. 4



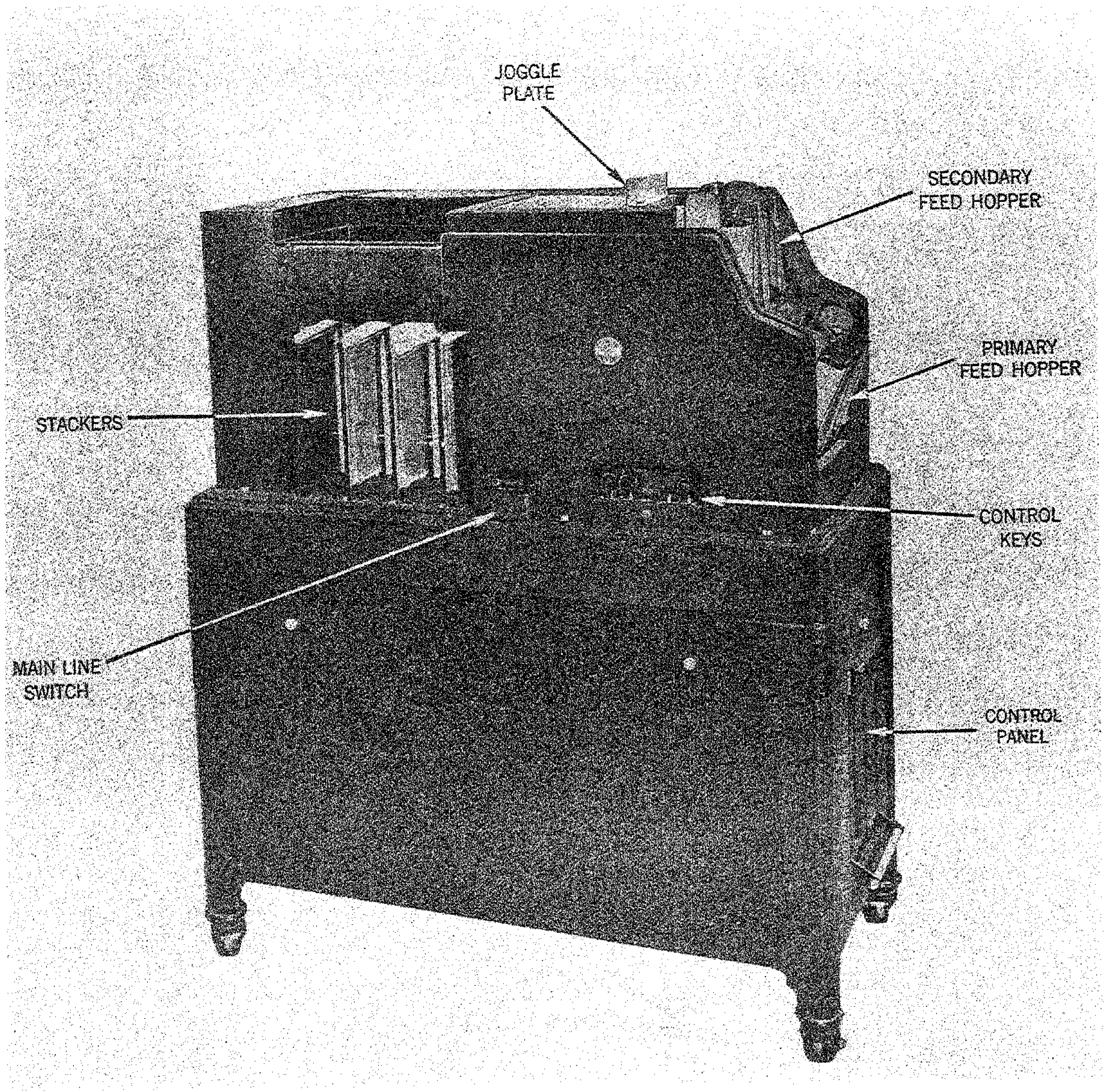
Sorter Operating Principle

FIG. 5



COLLATOR OPERATING PRINCIPLE

FIG. 6



IBM COLLATOR
TYPE 077

FIG. 7

The particular operations desired must be wired on a control panel such as the one shown in the lower right-hand corner of Fig. 7.

__Merging__: Very often it is necessary to merge two large groups of cards, each group being in numerical order, into a single group which is also in numerical order. This is done by placing one group of cards in the primary hopper and the other group in the secondary hopper and feeding the cards through the Collator, whose control panel is wired in such a way that the selector control unit operates to merge the two groups into a single group in numerical sequence. This operation could be done on the Sorter only if the numbers identifying the cards of each group happened to be in the same card columns, and even then, if large groups of cards are involved, it is very inefficient. For example, if 10,000 cards were in each group, the Sorter would have to sort 20,000 cards five different times while the Collator could do the job in a single operation. This means that approximately 3.7 hours would be required on the Sorter and only 36 minutes for the Collator.

__Matching__: The Collator can be wired to select from two groups the cards that have the same numbers, rejecting all other cards. For example, suppose there are several thousand cards in group "A" and group "B", each group being in numerical sequence. The Collator will compare the identifying number of the cards in group "A" with the identifying number of the group "B" cards and select from the two groups all of the cards whose numbers are the same, leaving both large groups in their original numerical order. This involves the use of

the four pockets shown in Fig. 6, two pockets for the matched cards and two pockets for the unmatched cards. By using the merging and matching operations of the Collator, it is possible to select certain cards from a large group, use these cards for a particular operation, and then return them without changing the numerical sequence of the original group.

Counting and Checking: The Collator can count cards from groups "A" and "B" and merge them according to a predetermined system. For example, the Collator could be used to get a single group of cards which would consist of group "A" cards with every tenth card a card from group "B". The checking operation of the Collator is used to check the sequence of a set of cards to insure that no card is out of order.

Reproducer.

The 519 End Printing Reproducing Punch is shown in Fig. 8. A schematic diagram of the reading and punching units of the Reproducer is shown in Fig. 9. Cards feed from the two card hoppers and are read by the four sets of brushes shown. The cards going through the right-hand side of the Reproducer travel underneath the punching dies, which punch holes on the cards corresponding to those read by either the reproducing or punch brushes shown in Fig. 9. The Reproducer is ordinarily used for reproducing, gang punching, mark sensing, summary punching, comparing, printing, and punching constant numbers.

Reproducing: The reproducing brushes of the reading unit will read the holes that are on the cards feeding through this unit and

TYPE 519 END PRINTING REPRODUCING PUNCH

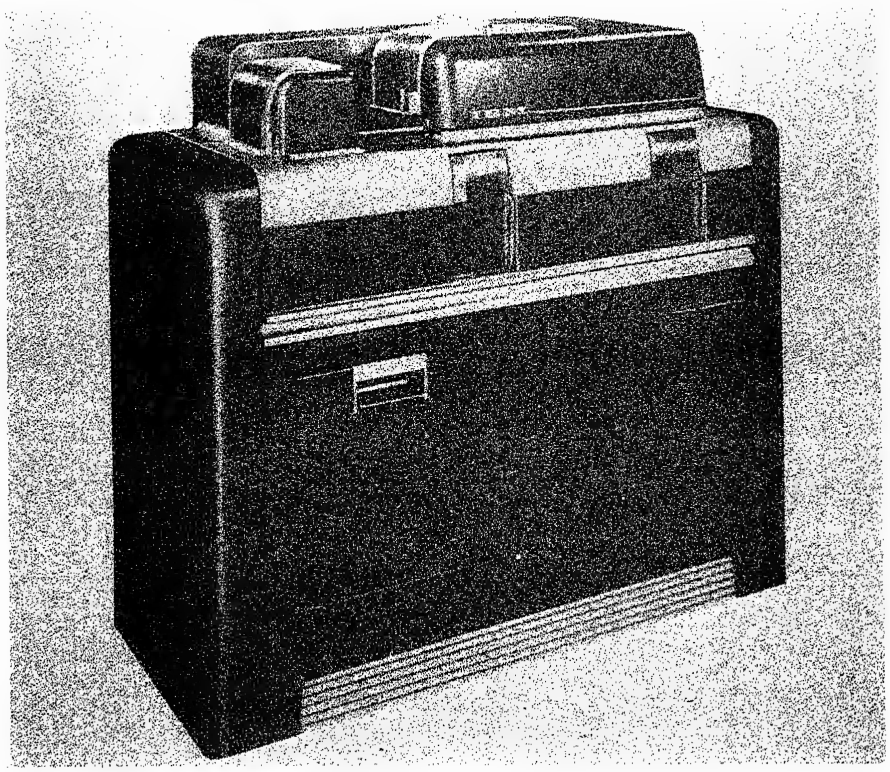
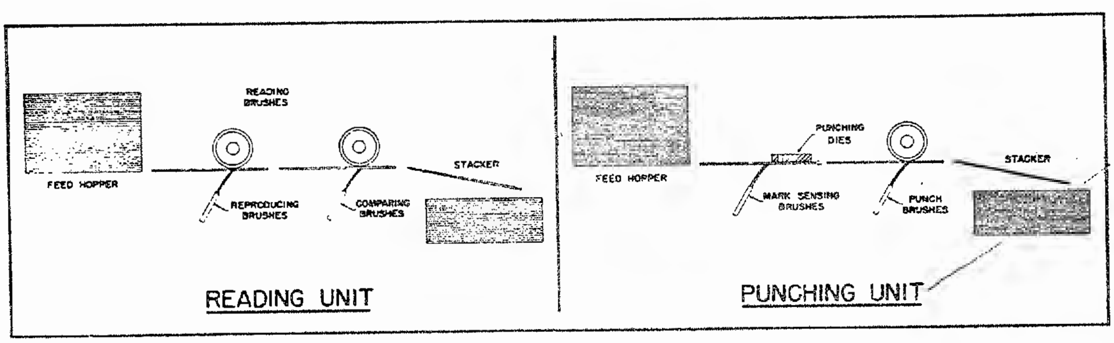


FIG. 8



SCHEMATIC DIAGRAM
FIG. 9

will activate the punching dies in order to punch corresponding holes on the cards feeding through the punching side of the Reproducer. Thus, dependent on the machine wiring, all or any part of the information punched on one set of cards can be punched in the same or different arrangement into another set of cards. During the reproducing operation, the Reproducer can be wired to compare the original number with the one punched and in this way effectively check the accuracy of its own operation.

Gang Punching: The operating of gang punching takes the information punched on one card and punches this same information on a number of cards which follow. It is possible to combine the reproducing and gang punching features to obtain a group of cards which are punched with information from the cards feeding through the reading unit and also with information that is on special cards mixed with the cards that are going through the punching side of the Reproducer.

Mark-Sensing: The mark-sensing brushes are shown in the punching unit of Fig. 9. These brushes sense heavy graphite pencil marks that are on the cards and from this sensing, cause the punching dies to punch the cards with the digits corresponding to the ones marked. Mark-sensing is an easy way to transfer numerical data to punch cards without the use of a key punch.

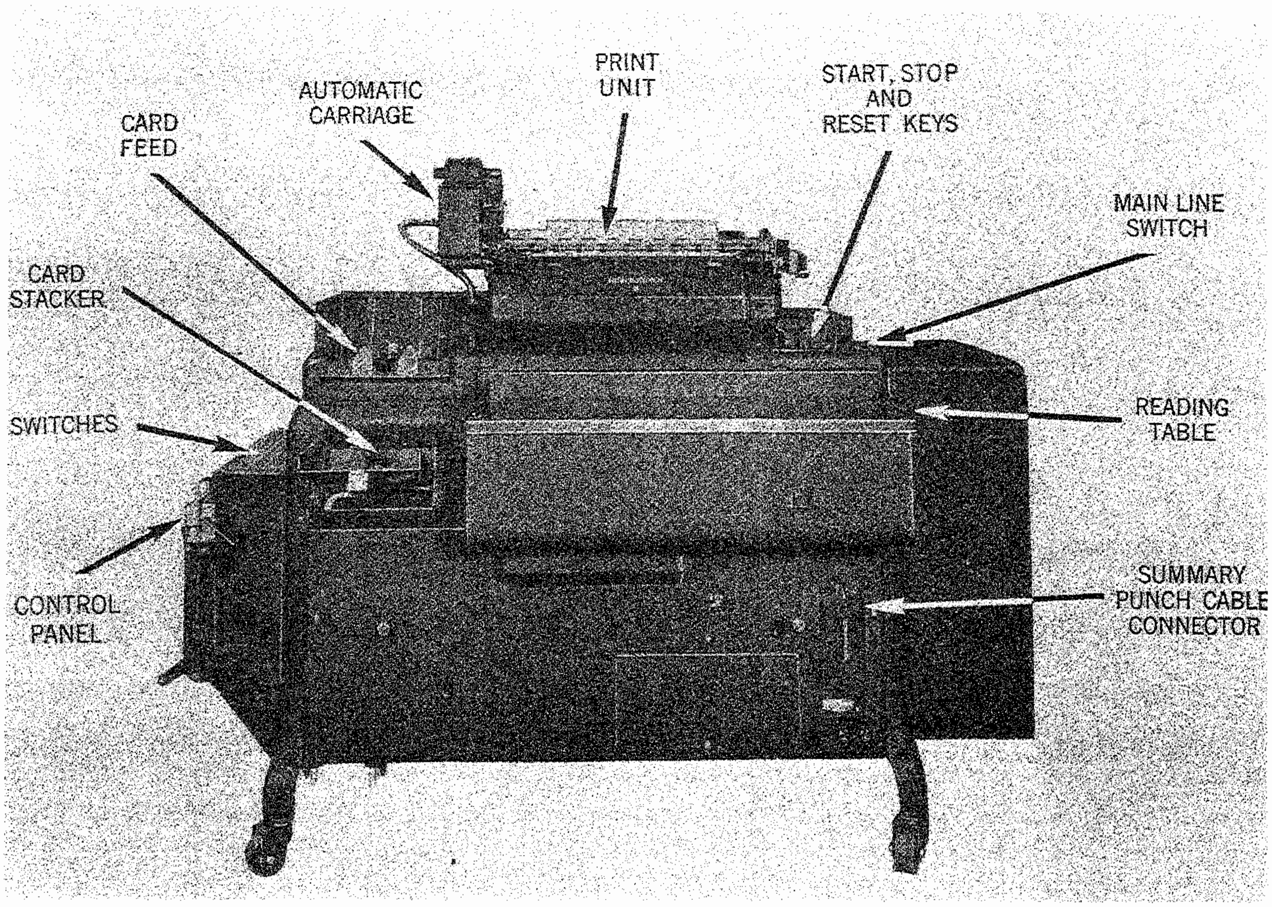
Miscellaneous Operations: The Reproducer can be used for various other operations. In summary punching, results developed in the Tabulator counters (see Tabulator section) are punched on cards feeding

through the Reproducer. The Tabulator and Reproducer are connected for this operation. For checking purposes, the comparing brushes of the reading unit and the punch brushes of the punching units can be wired to verify that both the reproducing and gang punching operations have been done correctly. This feature makes the Reproducer a very reliable machine. The Reproducer will print as many as eight digits across the face of a card fed through the punch side of the Reproducer (see Fig. 9). The digits to be printed can come from either the gang punching brushes (interpreting) or from the comparing brushes (transcribing). The Reproducer can be wired to emit numbers directly to the punching dies which means that a constant can be punched on a group of cards simultaneously with any of the operations previously mentioned.

Tabulator.

The Tabulator is ordinarily used to add and subtract numbers, summary punch, and to select and print both digits and letters. Numbers and letters can be printed directly from the IBM cards. Accumulated numerical results can be printed by the Tabulator or punched by the Reproducer directly from the various Tabulator counter groups. Figure 10 shows an alphabetical accounting machine, commonly referred to as a Tabulator.

Printing: The letters of the alphabet are punched in columns 1-26 of the IBM card shown in Fig. 1. These letters are formed by combining a zone punch (0, 11, or 12) with the digits 1-9. Figure 2 shows the upper and lower sets of brushes in the Tabulator card feed.



Alphabetical Accounting Machine

FIG. 10

The upper brushes read the zone impulse and the lower brushes read the digit impulse. The printing mechanism of the Tabulator senses these two electrical impulses, which together represent a letter, and combines them in such a way as to print the letter desired. The Tabulator has 88 type bars, one of which is shown in Fig. 11. In the left-hand drawing, the type bar is shown in its normal position. In the right-hand drawing, the type bar is shown after the zoning impulse (12) has caused the set-up pawl to enter the twelve notch and digit impulse (7) has caused the stop pawl to hold the type bars at the seven position. The hammer firing against the type bar has caused it to print the letter G. The digits 0-9 will print if the zoning impulse is omitted. The type 402 Tabulator will print letters and digits at the rate of 100 cards per minute. The type 417 Tabulator will print numbers only, but at the rate of 150 cards per minute. Thus the 402 can print 8,000 letters and digits per minute, while the 417 can print 12,000 digits in the same time.

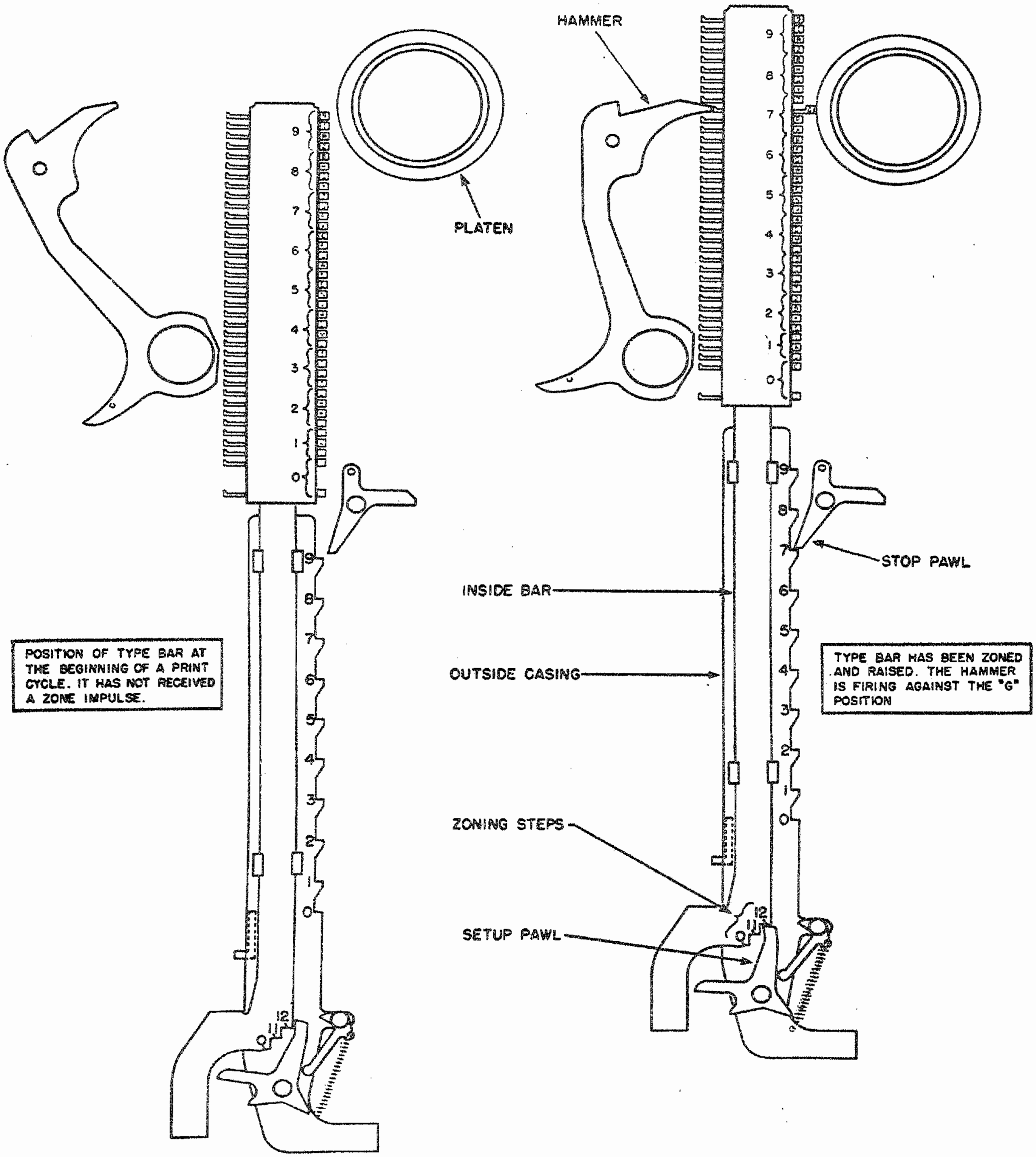
Counters: The Tabulator has 16 counter groups capable of storing up to 80 decimal digits. These counter groups can be coupled together to form various sized counters, the largest being an 80 decimal digit counter while the smallest would be a two-decimal digit counter. By using the upper brushes (shown in Fig. 2) to sense the sign of a number and the lower brushes to sense the number itself, the Tabulator can add and/or subtract numbers algebraically. Totals accumulated in the counter groups can be printed directly from these counters whenever desired.

Summary Punching: Summary punching was mentioned when discussing the Reproducer. The results developed in the counter groups just described can be punched on cards that are feeding through the Reproducer. This is accomplished by coupling the Reproducer to the Tabulator.

The IBM machines that have been described are standard units found in any IBM installation, doing either punch card accounting or computing work. In addition to these machines, each installation would have a Calculating Punch (commonly called a "Multiplier") to perform the basic arithmetic operations of addition, subtraction, multiplication, and division. The Computing Branch has had three different types of IBM Calculating Punches within a period of two years, each being a vast improvement over its predecessor. Incidentally, this is one of the advantages of the rental system which IBM employs, since machines can be traded in for improved models without a capital investment loss. Section two will describe the type 601 and type 602 Calculating Punches which the Computing Branch has used in the past, and the type 604 Calculating Punch that is in use at the present time.

CALCULATING PUNCH

A Calculating Punch is an IBM machine that will read numbers from a card, perform the particular arithmetic operation that is wired on the control panel, and then punch the results of this computation in different columns of the same card. Complicated mathematical equations



ALPHAMERICAL TYPE BAR

FIG. 11

can be solved by running a group of cards through the punch a number of times, each time wiring a different control panel and doing a different operation. The speed, storage and flexibility of the punch largely determine the size and complexity of the problems that can be successfully undertaken. The problems that can now be handled in the Computing Branch are much more complicated than the ones done on the original type 601 Calculating Punch.

601 Calculating Punch.

The 601 was the original Multiplier used in the computing installation at NOTS. This machine could perform the operations of addition, subtraction, multiplication, and punching, but could not find the quotient of two numbers. This meant that a card table of reciprocal values had to be used in order to find the reciprocal of the denominator in all division problems. The 601 Multiplier could then be used to multiply this reciprocal by the numerator in order to get the quotient desired. This was a slow and laborious means of computing a quotient. The operations of the 601 were performed by electro-mechanical means at the relatively slow average speed of nine cards per minute. This machine was so severely handicapped in both speed and flexibility that it was soon discarded by the Computing Branch and replaced by an improved type electro-mechanical IBM Multiplier.

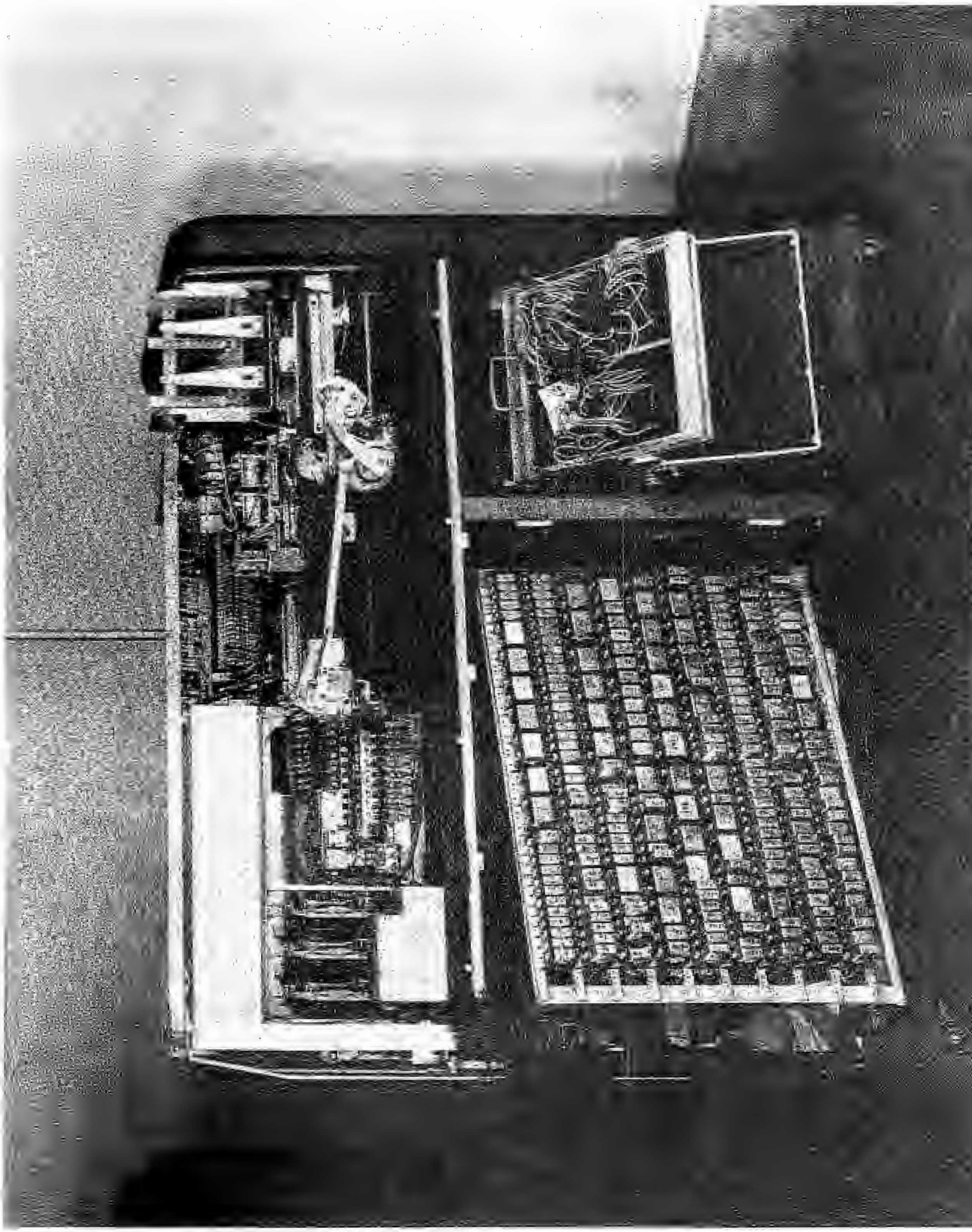
602 Calculating Punch.

The 602 was an improved model of the 601 Multiplier. Fig. 12 shows the 602 with the front cover plate removed. The card hopper

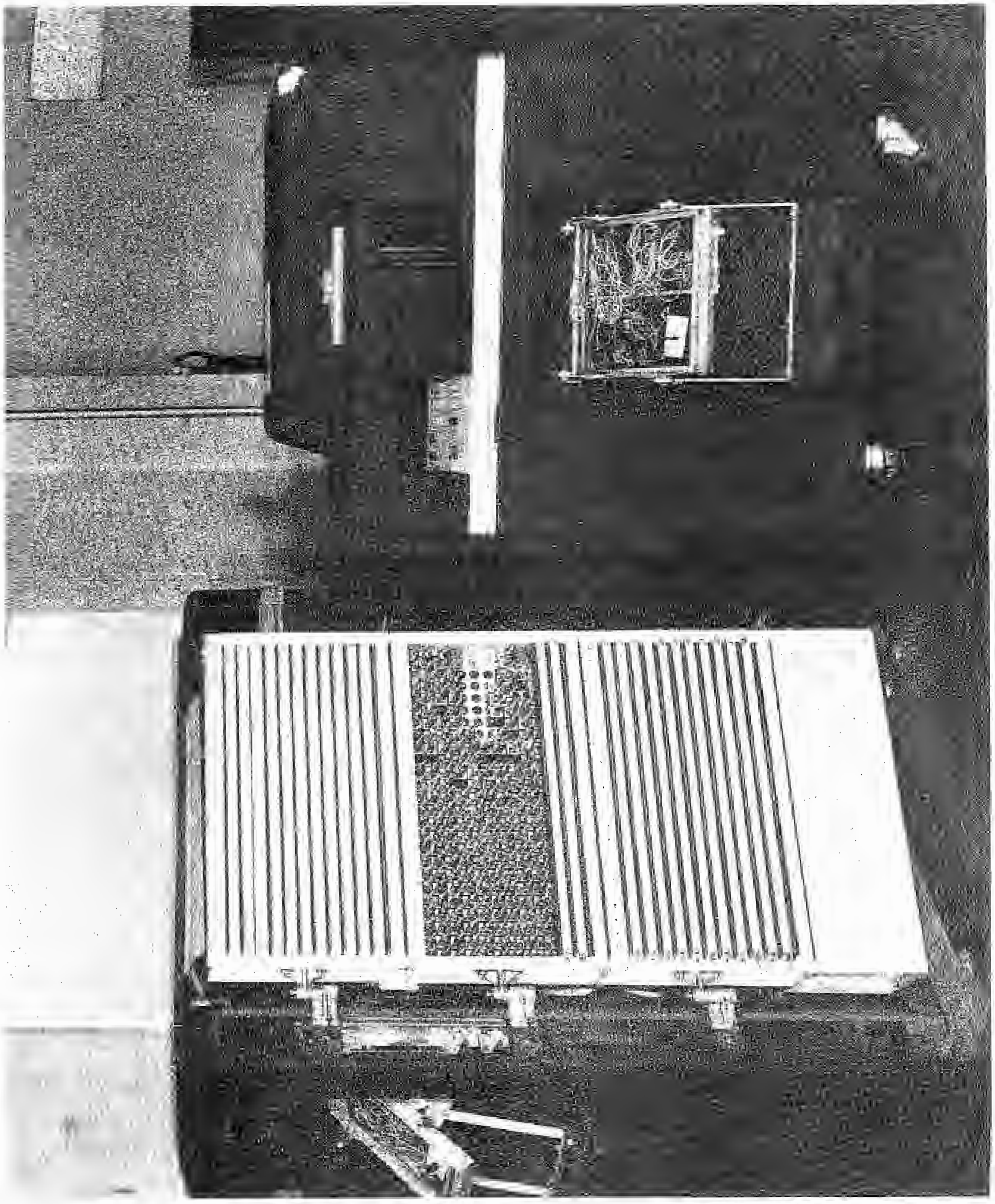
and mechanical feed are shown in the upper right-hand part of Fig. 12. The control panel and the relay racks are shown in the lower half of the same figure. The 602 Calculating Punch could perform simple or complex calculations involving addition, subtraction, multiplication and division, and punch the computed results on IBM cards. The number storage capacity of the 602 was considerably greater than that possible on the 601, and in addition, a limited amount of sequencing could be done. For example, an equation such as $P = \frac{A x b}{c} - d$ could be evaluated in one operation and the value of P punched on the same card from which the factors were read. The Multiplier was still electro-mechanical in operation and computed at an average speed of 20 cards per minute. The 602 was difficult to wire, slow in speed and limited in flexibility, which made its early replacement desirable. It was, however, a vast improvement over the 601 and a computer that could be used successfully in the computation of many different types of problems. An improved electro-mechanical Multiplier called the 602 A was soon put on the market as a replacement for the 602. The Computing Branch, however, went directly from the 602 to the 604 electronic multiplier and thus has never used a 602 A.

<u>604 Calculating Punch</u>.

The 604 Calculating Punch consists of an electronic 604 Multiplier and a Type 521 punch connected together (Fig. 13). The 521 reads the factors that are on cards, the 604 computes with these factors and at the end of the computation, the 521 punches out the result on the cards



602 CALCULATING PUNCH
FIG. 02



604 CALCULATING PUNCH
FIG. 13

from which the original factors were read. This means that numbers are continually going from the 521 to the 604 while computed results are travelling from the 604 to the 521. The interior of the 604 Calculating Punch is shown in Fig. 14. The basic components of this computer are the card feed, storage units, electronic counter, programs, selectors and card punch.

Card Feed: The punch cards pass through the 521 at the constant rate of 100 per minute. The reading brushes of the 521 read the factors from the cards and send these factors to the storage units of the 604.

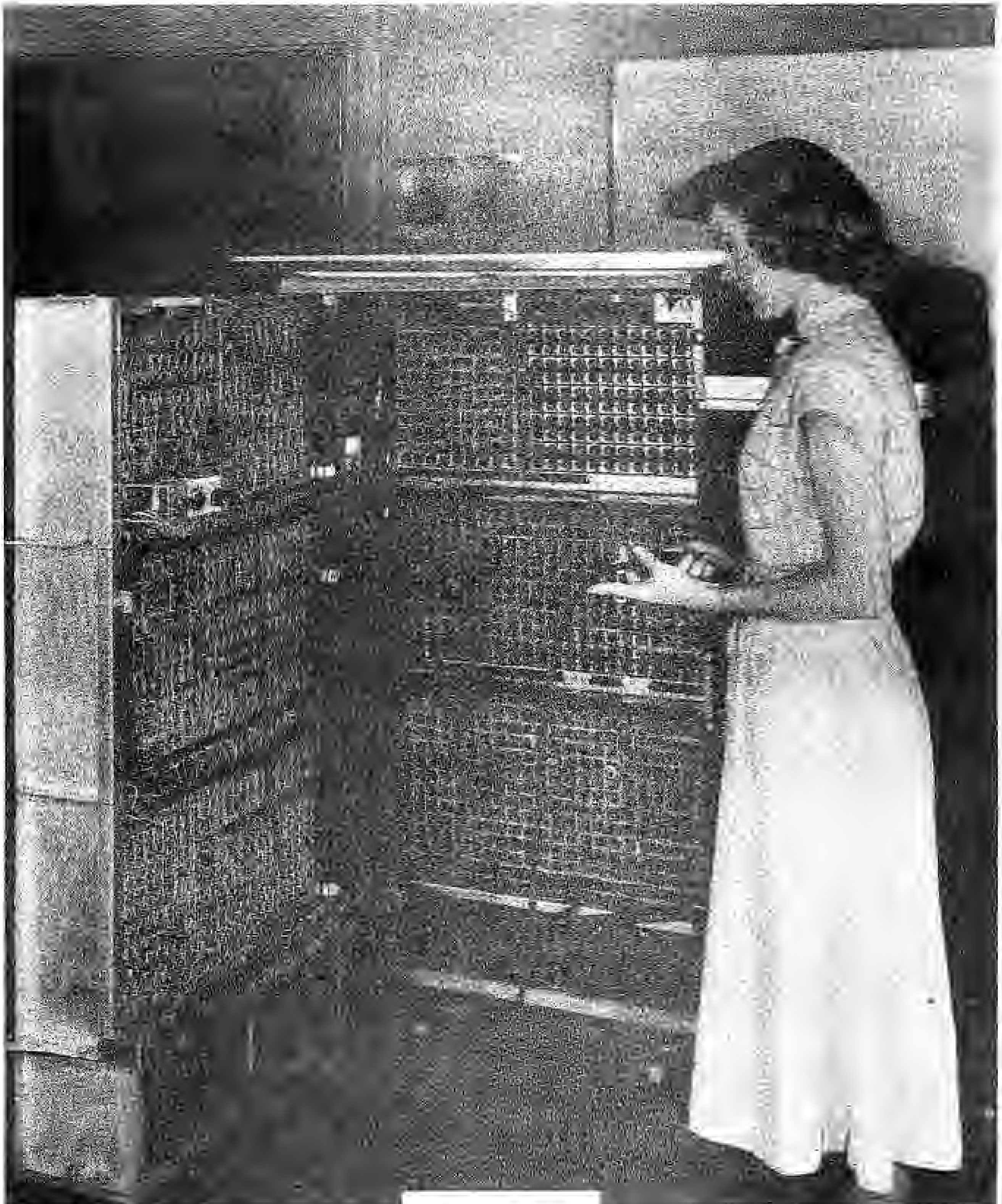
Storage Units: There are 37 decimal digits of electronic storage in the 604. This means that in a single operation the 521 could read from a card and transfer to the 604 a number of factors containing as many as 37 decimal digits. The 604 would use these numbers to perform the desired calculation and return the result to the 521 to be punched on the same card from which the factors were read. Sixteen digits of this storage may be used to either transfer factors into the 604 from the 521 or transfer results from the 604 to the 521 for punching. However, these storage units cannot be read into and read out of at the same time.

Electronic Counter: There is a thirteen position electronic counter in the 604 which will add, subtract, and shift numbers. Multiplication and division are accomplished by successive addition and

shifting or subtraction and shifting. The electronic counter is the only component that has the ability to add and subtract numbers, which means that all of the computations must be done in, or with the aid of, this counter. Numbers from the cards cannot be read directly into the counter, but final results may be punched from the counter.

Programs: For each card passing through the 521 there are 60 separate programs available in the 604. Each program can be wired to perform one operation. For example, Program 1 could be wired to take a number out of storage unit 1 and add it into the electronic counter. Program 2 could be wired to multiply the number in storage unit 2 by the number in the multiply quotient storage unit, thus developing the quantity AB+C in two program steps.

Selectors: The Calculating Punch has several different types of selectors which greatly add to its flexibility. The 521 has selectors with which to control its wiring and thus on one card the punch might read numbers from columns 12-30 and punch the results in columns 30-45 but on the following card, by using these selectors and control punches in the card, numbers could be read from columns 50-70 and results punched in columns 70-80. There is a second type of selector that has its pick-up on the 521 and its points in the 604. This selector is used to select the operation that the 604 will carry out on any one card cycle. Selections can also be made during the calculate cycle of the 604 since the computer can recognize whether or not a number is positive, negative or zero on any one of the sixty programs, and from these



604 MULTIPLIER
FIG. 14

conditions, determine the subsequent calculations to be performed.

Card Punch: When the 604 finishes calculating, the results are stored in the electronic counter or sixteen positions of the storage units. These results are then punched by the 521 at the same time that new factors are reading into the 604 for the next operation. Results are computed and punched at a constant rate of 100 cards per minute.

The 604 Electronic Calculating Punch is a vast improvement over the old 601 and 602 electro-mechanical models. Many problems can be easily solved on the 604 which would have been completely impractical on the mechanical multipliers.

The previous discussion has described the basic punch card machines needed to make up a computing installation built around the 604 Electronic Calculating Punch. This facility is especially well suited to problems which can be computed in a step-at-a-time manner, and has been used extensively at NOTS. The following section will discuss the manner in which this type of computing installation can be used to solve complicated computational problems.

PROBLEMS SUITED TO A TYPE 604 COMPUTING INSTALLATION

The 604 is best suited to solving problems in what might be termed a "shotgun" approach. That is, it is most efficient when used to solve a large number of problems of the same type concurrently, where step 1 is carried out for all the problems, then step 2, etc., until the final answers are obtained for all the problems together. This is in distinction to other types of computers which prefer a

serial mode of operation, in which all the steps are carried out for the first problem, then the second is solved by repeating the steps, etc. Some problems, of course, have a sufficiently brief and simple sequence of steps to be solvable in a single 604 operation. Examples of both types of problems are given in the following paragraphs.

<u>Typical Problems That Can Be Solved in a Single Operation on the 604.</u>

<u>Numerical Integration</u>: The 604 is especially adept at solving numerical integration problems using Simpson's Rule for the summation of the ordinates. In one operation the 604 will compute $\int_a^b F(x)\,dx$ using values of $F(x)$ which would ordinarily have been previously computed on the 604. The cards pass through the 604 at the rate of 100 cards per minute, with the final value of the integral punched on the last card. The 604 will automatically take care of the weighting factors by which ordinates are to be multiplied, take care of the correction for the first and last ordinate, and will automatically take care of any number of changes in Δx. The only restriction is that changes in Δx must occur after an even number of intervals, since only then is Simpson's formula valid. The fact that a change in Δx is automatically taken care of by the 604 allows the mathematician to compute fewer values of the integrand and still maintain a specified accuracy. Formerly, it has sometimes been easier to compute additional values of the integrand than to introduce frequent interval changes.

<u>Statistical Problems</u>: The 604 multiplier can be used conveniently

to compute the variance and mean of a variable or the co-variance and means of two variables at the rate of 100 values per minute, each value having been previously punched on an IBM card. For example, the 604 would require only 20 minutes to compute two means, the variance, and the co-variance of two variables for 1,000 terms in each summation. This speed allows the statistician to undertake problems too laborious to be solved on a desk computer.

Evaluation of a Function Using Table Cards: Punch card tables are available, or can be made up, for almost any function desired. A table consists of a number of cards, each card having a value of the argument, the function value for this argument, and usually the first or the first and second differences of the function. The interval of the argument and the order of the differences determine the accuracy that can be obtained from a given table. The Sorter, Collator and the 604 are all used in evaluating a function with the aid of table cards. This method is satisfactory if there are a large number of values of the Function to be determined and if the table is not too large and cumbersome.

Evaluation of a Function Using a Series Approximation: Any convergent series can be evaluated on the 604 by considering one term at a time and then summing these successive terms. This is an extremely laborious process since each term might require a separate IBM card and the number of terms could be very large. Fortunately, the 604 has a feature called program repeat which causes it to activate the sixty programs over and over again until some condition (zero balance,

plus balance, or negative balance) in the problem causes this repeating to stop. By using this feature, such functions as sin x, cos x, e^x, e^{-x^2}, sinh x, cosh x and log x can be evaluated by a series summation using a single IBM card. For example, if a number of arguments are punched on IBM cards the 604 will, if wired to perform this operation, compute sin x to seven decimal places at the approximate speed of 40 values per minute.

Evaluating Complex Functions: Practically any complex transcendental function can be evaluated on the 604, the main requirement being that there are enough solutions required to make it economical to wire up the 604 boards necessary to solve the problem. The following integral was evaluated on the 604 multiplier:

$$\int_c^m \frac{D(a) - D(m)}{\sqrt{m^2 - a^2}}\, da$$

where

$$D(a) = \frac{(a-c)}{a^2 e^2}\left[\frac{a+c}{a}\left(\sqrt{a+a^2} - a\right) - \frac{1}{2}\,\frac{(a-c)}{\sqrt{a+a^2}}\right]$$

and

$$D(m) = \frac{(m-c)}{m^2 e^2}\left[\frac{m+c}{m}\left(\sqrt{m+m^2} - m\right) - \frac{1}{2}\,\frac{(m-c)}{\sqrt{m+m^2}}\right]$$

$c \leqq m$	m was taken in steps of .05
$o \leq m \leq 1$	c was taken in steps of .10
$o \leq c \leq 1$	4 significant figure accuracy was desired.

The integrand had to be evaluated for a large number of values of "a" for each change of the parameters "c" and "m" . The various values of "a", "c" and "m" were put on separate cards. By doing one step at a time, the value of all of the integrands (several thousand) were finally computed. In this type of procedure the final answers are all computed in the last run through the 604.

<u>Data Reduction Problems</u>: The common types of data recording instruments, such as (1) the Bowen camera, (2) Theodolite, (3) Hyperbolic-Doppler, and (4) SCR-584 Radar, gather data which can be reduced on a 604 computing installation to give position, velocity and acceleration. Reduction of Theodolite data, for example, has been carried out on IBM equipment for several years. However, the equations used for this reduction have changed, becoming more and more complex as the computing equipment improved. A method formerly used on the 604 Multiplier required that the following equations be evaluated:

$$A_{12}=\sin H_1 \sin H_2 + \cos H_1 \cos H_2 \cos (S_1-S_2)$$

$$A_{13}=\sin H_1 \sin H_3 + \cos H_1 \cos H_3 \cos (S_1-S_3)$$

$$A_{23}=\sin H_2 \sin H_3 + \cos H_2 \cos H_3 \cos (S_2-S_3)$$

$$B_1= (2X_1-X_2-X_3)\cos H_1 \sin S_1-(2Y_1-Y_2-Y_3) \cos H_1 \sin S_1+(2Z_1-Z_2-Z_3)\sin H_1$$

$$B_2=-(2X_2-X_1-X_3)\cos H_2 \sin S_2-(2Y_2-Y_1-Y_3) \cos H_2 \sin S_2+(2Z_2-Z_1-Z_3)\sin H_2$$

$$B_3=-(2X_3-X_1-X_2)\cos H_3 \sin S_3-(2Y_3-Y_1-Y_2) \cos H_3 \sin S_3+ (2Z_3-Z_1-Z_2)\sin H_3$$

$$-2\gamma_1 + A_{12}\gamma_2 + A_{13}\gamma_3 = B_1$$

$$A_{12}\gamma_1 - 2\gamma_2 + A_{23}\gamma_3 = B_2$$

$$A_{13}\gamma_1 + A_{23}\gamma_2 - 2\gamma_3 = B_3$$

H_1 and S_1 are the elevation and azimuth angle of the first Theodolite station whose co-ordinates are X_1, Y_1 and Z_1. The quantity γ_1, is the straight-line distance between the first Theodolite station and the rocket. An additional number of arithmetic operations were required in order to change each value of γ back to the Cartesian co-ordinates of the launcher.

Twelve different 604 boards were wired up, each board doing a particular part of the solution, so that by using the 604 boards in sequence, along with certain other IBM machines, the final answers could be computed relatively quickly. This complicated procedure was feasible because: (1) the computation of each individual point was independent of the value of the other points, (2) there were an economically sufficient number of points (on the average 100) to compute for each missile, and (3) the procedure could be standardized for all Theodolite problems. This same problem was computed at one time on the 602 Multiplier and required 42 different 602 boards to compute the final answers.

General Types of Problems which can be Solved on the 604.

A number of problems have been discussed, all of which could be solved on the 604. These problems included numerical integration,

statistical procedures, evaluating a function using table cards, evaluating a function by series approximation, evaluating a complex function, and data reduction. In addition there are a large number of other problems which can be computed successfully on a 604 type computing installation. These include: (1) table construction, (2) optical ray tracing, (3) determining the thermodynamic properties of dissociating combustion gases, (4) certain types of differential and partial differential equations, (5) solutions of linear, non-linear and transcendental equations, and (6) interpolation.

The preceding sections described the various types of IBM machines that would make up a computing facility built around the 604 Calculating Punch. Fig. 15 shows the IBM installation of the Computing Branch at NOTS prior to the delivery of a Card Programmed Calculator (CPC) and corresponds to the type of installation described in the previous sections. In the lower left-hand corner, Fig. 15 shows the 604 and 521 connected to form a Calculating Punch, then a Reproducer and a Tabulator. In the lower right-hand corner, two Type 602 Multipliers, a Sorter, and a Collator are shown. The Key Punch is near the Sorter, but not in view.

Many problems cannot be solved on this type of facility, and other problems, some of which have been mentioned, can be computed but not easily, speedily, or economically. The 604 is severely limited in storage space and sequencing ability when compared with a general pur-

pose selective sequencing digital computer. To overcome these deficiencies, the IBM Corporation developed the CPC and made first deliveries in November of 1949. The CPC is a card-sequencing digital computer capable of solving many complex computational problems. The Computing Branch at NOTS has had two Card Programmed Calculators since February, 1950, and for many months was the only installation in the country operating two of these computers.

At the present time, the Computing Branch of the Mathematics Division at NOTS has the following IBM installation:

(1) The standard IBM machines previously mentioned, consisting of two Key Punches, two Reproducers, one Sorter and one Collator.

(2) One 604 Calculating Punch

(3) One standard Card Programmed Calculator

(4) One improved Card Programmed Calculator

With this installation, the 604 computing previously discussed can be done, and in addition, problems which lend themselves more readily to computation on the CPC can also be handled. Whether a problem should be done using the "shotgun" technique of the 604, or the sequencing approach of the CPC, must be determined individually, but for most problems, the CPC has a distinct advantage. The remainder of this report will be devoted to describing the CPC, outlining its advantages and giving examples of the problems which have been, or readily can be solved, using either the standard or the improved model.

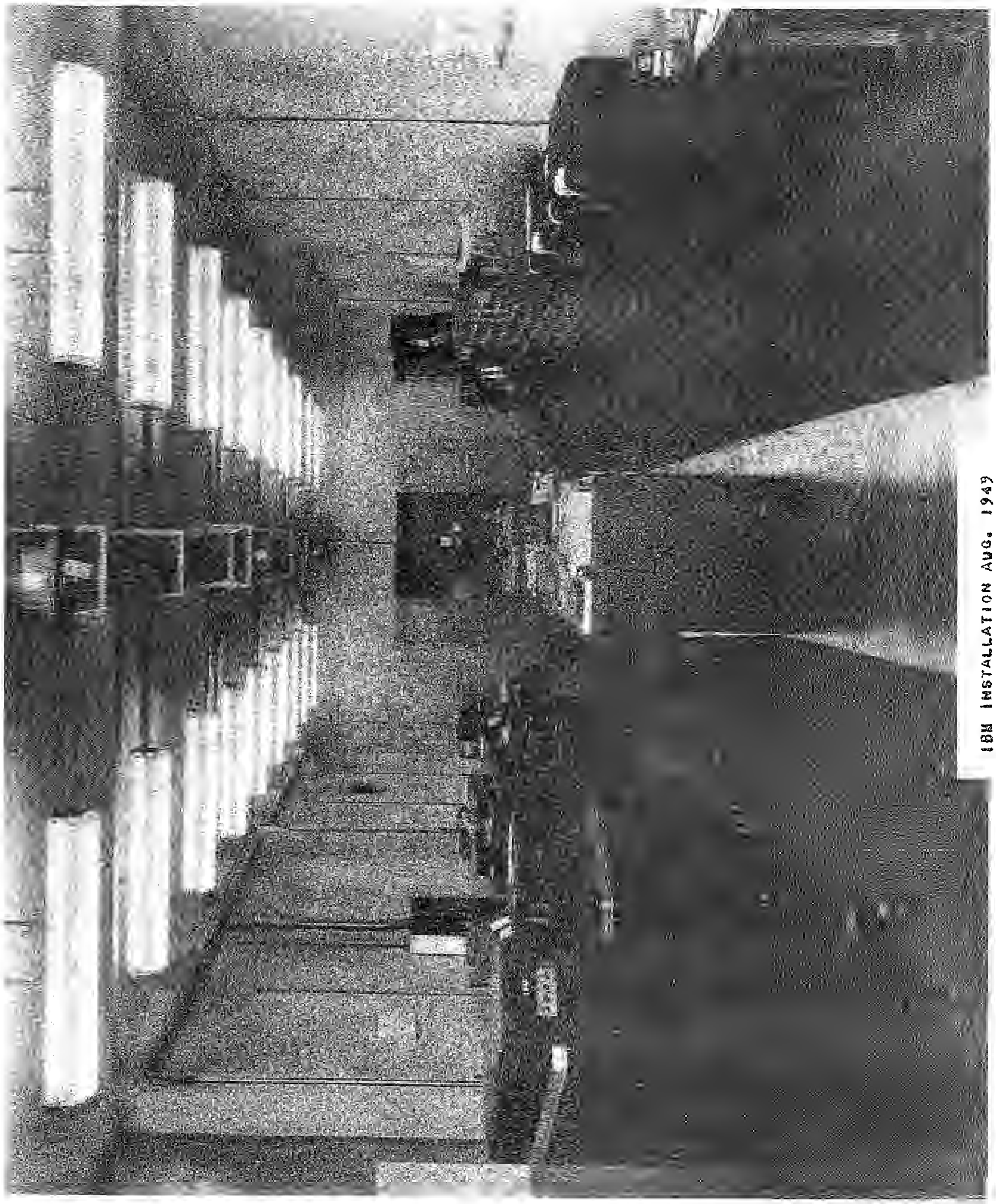


Fig.15

CARD-PROGRAMMED CALCULATOR

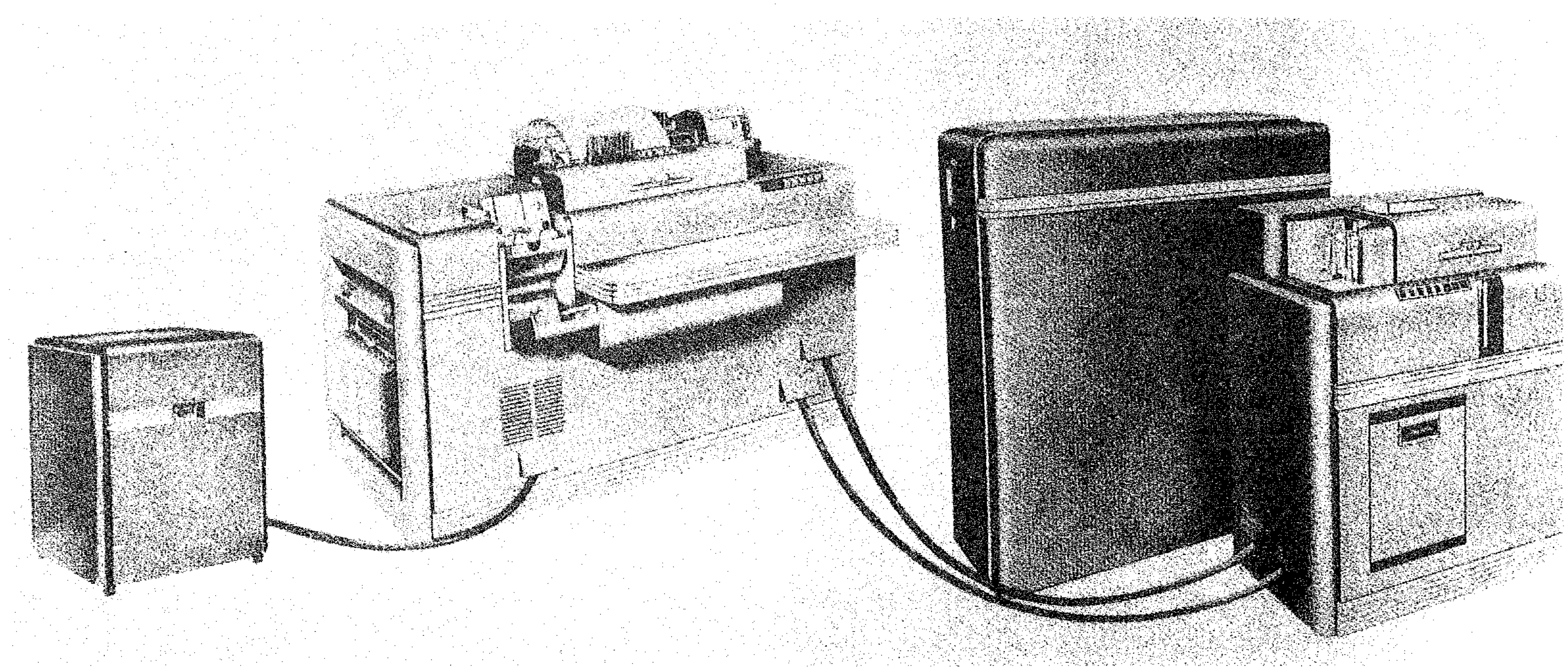
The Standard CPC.

This unit is made up of three standard IBM machines and one or more special storage devices designed for use with the other components. Fig. 16 shows the four units and their connecting cables. The CPC consists of a Tabulator, a 604 Multiplier, an end punching Reproducer, a number storage device and special electrical circuits to connect these components together. The individual units of the computer and their function in relation to its overall operation will be outlined in the paragraphs that follow.

The 604 Multiplier: The 604 Multiplier does the major portion of the actual computing, and consists of a standard 604 with certain auxiliary circuts which enable it to be used as part of the CPC. There are three channels in the CPC each capable of transferring a ten-digit number. Two of these channels are used to introduce numbers into the 604, and the remaining one is used to transfer results from the 604 to some type of storage. All operations that are necessary for a particular problem must be wired on the 604 board and would ordinarily be made up of such things as addition, subtraction, multiplication, division, square root, sin x, cos x, e^x, cosh x, log x, and any special operation that is used repeatedly, such as $(1-x^2)$. A selection of the particular operation to be performed is controlled by the instruction card passing through the Tabulator and can be any one of the oper-

ations that is wired on the 604 board. For example, card one might compute $A \cdot B$, card two A/B, and card three $\sqrt{A}$. A limiting feature of the standard CPC is that only sixty programs are available in the 604. If the problem requires several of the more difficult functions, such as square root, sin x and log x, then it must be done in two or more individual steps since these functions cannot all be included in a single 604 board.

Tabulator: This machine, like the 604 Multiplier, is a standard Tabulator with certain auxiliary circuits to enable it to operate as part of the CPC. The Tabulator does all of the card sequencing and is thus the master machine of the group. It reads the instruction cards of a problem and translates the numbers on the instruction cards into commands, telling the other components what to do on any particular card cycle. The Tabulator has six ten-digit counter groups and one seven-digit counter group that will add, subtract, accumulate, store, and transfer numbers. This allows the Tabulator to be used for accumulating and storing partial results in a time saving manner. For example, the Tabulator could be used to accumulate $\frac{A}{B} + CD - F$ as follows: Develop the quotient in the 604 on card number one, and send it to a Tabulator counter for storage, develop the product on card number two, sending it to the same counter to be added and stored, on the third card, subtract F from this same Tabulator counter group, thus accumulating the function $\frac{A}{B} + CD - F$ with three instruction cards. The numbers A, B, C, D, and F enter the 604 via its two input channels and can originate from the instruction cards, other Tabulator counter groups, or from the Type 941 storage.



Type 941 — Type 402-417 — Type 604 — Type 521

IBM Card-Programmed Electronic Calculator

The Tabulator is one of the two output devices of the CPC. It can print the numbers that are in the three ten-digit channels on any card cycle, print from any one of its seven counter groups, and in addition print from any column of the card itself. This high speed printing ability allows the mathematicians a wide latitude in printing out individual operations, partial results, or final results in report form.

Storage Counters: The original CPC contained 16 storage counters each capable of storing a ten-digit number. Each CPC now in operation in the Computing Branch has three Type 941 Storage Units connected in series to give 48 ten-digit storage counters instead of the standard 16. This greatly increases the power and flexibility of the computer and enables it to solve conveniently problems which require considerable storage capacity. Each one of the 48 Type 941 Storage Counters is mechanical in operation and can store a number either positively or negatively, but cannot add or subtract two numbers. A number must enter a counter group through the 604 exit channel and once entering, the number is stored until called for by a Tabulator instruction card, at which time it goes to the 604 over either of the two input channels. Two numbers can be selected from storage and a partial result sent to storage all on the same card cycle.

Type 521 Punch: The Type 521 Unit will punch directly from the 604 or from any one of the seven Tabulator counter groups. This punch card output is used extensively for storing final results and for punching cards with partial results when a problem exceeds the capacity of the

CPC. These partial results are read back into the CPC at a later point in the problem in order to compute the final results. Inverting a large matrix is an example of a problem that must be computed in this manner. The Type 521 Punch does nothing but punch cards when used as a part of the CPC. It no longer reads factors from the cards or controls the operation of the 604 as it does when the two are connected together to form a Calculating Punch.

CPC Operation: The preceding discussion outlined the function of the component parts of the CPC. These units combine to form a versatile, small scale, automatically sequencing, digital computer. This computer operates at the rate of 150 cards per minute, each card containing instructions, numerical data or both instructions and numerical data. On any one card cycle the computer can do one or a series of numerical operations. That is, on one card the computer could multiply A times B or it could evaluate sin x to seven decimal places by summing the infinite series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - - -$. The computation of sin x would require only one instruction card but would take two or three card cycles, each card cycle requiring four tenths of a second. The CPC has 55 storage counters in which constants, partial results and final results can be stored. Seven of these counters will add, subtract, and accumulate. Results or partial results can be printed on paper from the Tabulator or punched on cards, directly from the 604 or from any of the seven Tabulator counters.

To solve a problem on the CPC three boards must be wired, a Type 604 board, a Tabulator board and a Type 521 board. Fortunately, general purpose boards can often be wired which will solve a number of different problems. After the boards have been wired to do the operations desired, a deck of instruction cards must be prepared. Each instruction card causes the CPC to perform a particular arithmetic operation and a deck of these instruction cards will compute a problem in a sequential manner one step at a time, storing partial results until the final answer is printed or punched at the end of the computation. The size of the instruction deck depends entirely on the problem being solved and has in the past run as high as 2000 cards before a final answer is computed.

CPC - MODEL II.

Experience gained through operating the standard CPC soon indicated that many problems could not be completed in a single step, because there were an insufficient number of 604 programs to wire up the necessary operations. In August, 1950, the Computing Branch requested permission from IBM to build a switching device for use with the CPC. This device would have added to the capacity and flexibility of the Computer. Instead of granting permission to use an auxiliary device with their machines, (all IBM machines are rented), they offered to build a special 604 that would incorporate the features desired and offer other advantages in addition. This improved 604 was delivered just four months after the original proposal was made and is now oper-

ating as part of the CPC. This Computer is the only one of its kind in operation at the present time and has proven itself to be far superior to the Standard CPC.

Model II 604. The improved model of the CPC is the same in every way except for the new Type 604 and several simple auxiliary circuits in the Tabulator. The major advantage of this 604 is that it has 180 programs instead of the usual 60. This means that a Model II 604 board can now be wired to add, substract, multiply, divide, square root, and find such functions as sin x, cos x, e^x, cosh x, sinh x, in addition to several other operations that each problem might require. This greatly increases the computational power of the Computer and enables it to solve problems which would have to be done in several separate steps on the old CPC.

The Model II, 604 has 10 additional selectors, a new type of program repeat circuit, uni-directional plug hubs, and several other improvements which all combine to make it a definite improvement over the standard model. For example, problems have been computed in a single operation on the improved CPC which required three separate involved steps on the Standard Model. The computing speed in this case was increased approximately four times.

Problems Done on the CPC.

Several examples of typical CPC problems will be outlined in order to indicate the adaptability of both the standard and improved models

in obtaining many complex numerical solutions. The first example is a problem in optics and can be computed on either CPC. The second is a Theodolite reduction problem which can be done in one continuous operation only on the improved CPC.

Optical Ray Tracing: A very general solution to the problem of optical ray tracing has been developed. This system will trace both ordinary and skew rays through a centered or non-centered optical system of ordinary lens elements, plain surfaces, prisms, and reflecting surfaces. This makes ray tracing on the CPC applicable to practically any optical system that can be devised. In addition, an iterative method has been developed which will handle ray tracing through aspheric surfaces, making the CPC ray tracing system very powerful and completely general. The following equations must be solved once for each surface through which the ray is traced:

Initial Data.

Vx, Vy, and Vz are the components of the Vector $\vec{V}$ which has its origin at the intersection of the axis with a particular lens surface and terminates at the intersection of the optical ray and this same lens surface. Qx, Qy, and Qz are the direction cosines of $\vec{V}$.

Interface Data.

A, B and C are the direction cosines of the line connecting the poles of two optical elements, D is the separation of two elements, C is the reciprocal radius of curvature of a surface, N is the refractive index of a lens. The subscript k keeps the initial data and the in-

terface data of the elements of the system in the correct notational order.

$$\vec{Q}_{k-2} \circ \vec{U}_{k-1} = Q_{(k-2)x} A_{k-1} + Q_{(k-2)y} B_{k-1} + Q_{(k-2)z} C_{k-1}$$

$$\vec{Q}_{k-2} \circ \vec{V}_{k-2} = Q_{(k-2)x} V_{(k-2)x} + Q_{(k-2)y} V_{(k-2)y} + Q_{(k-2)z} V_{(k-2)z}$$

$$M_{k-1} = d_{k-1}\left(\vec{Q}_{k-2} \circ \vec{U}_{k-1}\right) - \left(\vec{Q}_{k-2} \circ \vec{V}_{k-2}\right)$$

$$\vec{F}_k \circ \vec{U}_{k-1} = \left(\vec{U}_{k-1} \circ \vec{V}_{k-2}\right) + M_{k-1}\left(\vec{Q}_{k-2} \circ \vec{U}_{k-1}\right) - d_{k-1}$$

$$F_k^{\ 2} = \left(\vec{V}_{k-2} \circ \vec{V}_{k-2}\right) + d^2_{k-1} - M^2_{\ k-1} - 2d_{k-1}\left(\vec{U}_{k-1} \circ \vec{V}_{k-2}\right)$$

$$a_k = \left[\left(\vec{Q}_{k-2} \circ \vec{U}_{k-1}\right)^2 - C_k\left(C_k F_k^{\ 2} - 2\vec{F}_k \circ \vec{U}_{k-1}\right)\right]^{\frac{1}{2}}$$

$$P_{k-1} = C_k F_k^{\ 2} - 2\left(\vec{F}_k \circ \vec{U}_{k-1}\right) / \left(\vec{Q}_{k-2} \circ \vec{U}_{k-1}\right) + a_k$$

$$L_{k-1} = M_{k-1} + P_{k-1}$$

$$V_{kx} = V_{(k-2)x} + L_{k-1} Q_{(k-2)x} - d_{k-1} A_{k-1}$$

$$V_{ky} = V_{(k-2)y} + L_{k-1} Q_{(k-2)y} - d_{k-1} B_{k-1}$$

$$V_{kz} = V_{(k-2)z} + L_{k-1} Q_{(k-2)z} - d_{k-1} C_{k-1}$$

$$\nu_k = \frac{N_{k-1}}{N_{k+1}}$$

$$a_k' = \left[1 - \nu_k^{\ 2}\ (1 - a_k^{\ 2})\right]^{\frac{1}{2}}$$

$$g_k = a_k' - \nu_k a_k$$

$$Q_{kx} = \nu_k \, Q_{(k-2)x} - g_k (C_k V_{kx} - A_{k-1})$$

$$Q_{ky} = \nu_k \, Q_{(k-2)y} - g_k (C_k V_{ky} - B_{k-1})$$

$$Q_{kz} = \nu_k \, Q_{(k-2)z} - g_k (C_k V_{kz} - C_{k-1})$$

$$Q^2_{kx} + Q^2_{ky} + Q^2_{kz} \equiv 1 \text{ is a partial check.}$$

The computed components of $\vec{V}_k$ and $\vec{Q}_k$ are used as the initial data in the computation of $\vec{V}_{k+2}$ and $\vec{Q}_{k+2}$ at the next surface. In this manner a light ray is traced through an entire optical system.

The CPC is being used at the present time to evaluate and redesign a very complex system consisting of seventy optical surfaces. To trace a single ray through this system requires that the above equations be solved seventy times. This takes approximately twenty minutes on a CPC as contrasted to a week or more on a desk machine. To date the CPC has saved approximately two man-years of hand computing on this problem alone.

<u>Theodolite Data Reduction Problem</u> The usual Theodolite problem consists of trying to determine the most likely position of a missile from three lines of sight that do not ordinarily intersect. Each line of sight is determined from data recorded on a film by a particular Theodolite. This film is measured, and from these measurements the azimuth and elevation angles of the Theodolite are computed. A least squares solution is used to determine the coordinates of the missile, and from these coordinates the angular residuals are computed. A solution is computed at approximately 150 points for each firing.

The azimuth and elevation angles A and E must be evaluated three times for each point or 450 times on an average firing.

The quantities S, E, ΔS, ΔE, X_p and Y_p are functions of time and thus change with each point. The other quantities are a function of the Theodolite stations and are constant for any one trajectory. To solve for the azimuth and elevation angles A and E, the following equations must be evaluated.

$$A = A' + d \sin A' \tan E' + \theta$$

$$E = E' + d \cos A' - \frac{\left[(X_p - X_o)C_3\right]^2 \tan E'}{114.6} - 2.22 \times 10^{-7}\sqrt{(x-X_i)^2 + (z-Z_i)^2}\,(1-.0106E')$$

where

$$A' = S + \Delta S C_1 + \left[(X_p - X_o)C_3 + C\right] \sec E' \left[1 + \frac{(Y_p - Y_o)C_4 \tan E'}{57.296}\right] + e \cos(S_o + \phi) + O_s$$

and

$$E' = H + \Delta H C_2 + (Y_p - Y_o)C_4 + O_e$$

The values of A_1, A_2, A_3, E_1, E_2 and E_3 are used in the following equations to compute the x, y and z coordinates of the missile.

$$A_{11}x + A_{12}y + A_{13}z = C_1$$

$$A_{12}x + A_{22}y + A_{23}z = C_2$$

$$A_{13}x + A_{23}y + A_{33}z = C_3$$

where

$$A_{11} = \sum_{i=1}^{3} \frac{(1-a_i^2)}{P_i^2} \qquad A_{12} = -\sum_{i=1}^{3} \frac{a_i b_i}{P_i^2}$$

$$A_{22} = \sum_{i=1}^{3} \frac{(1-b_i^2)}{P_i^2} \qquad A_{13} = -\sum_{i=1}^{3} \frac{a_i c_i}{P_i^2}$$

$$A_{33} = \sum_{i=1}^{3} \frac{(1-c_i^2)}{P_i^2} \qquad A_{23} = -\sum_{i=1}^{3} \frac{b_i c_i}{P_i^2}$$

$$C_1 = \sum_{i=1}^{3} X_i \frac{(1-a_i^2)}{P_i^2} - \sum_{i=1}^{3} Y_i \frac{a_i b_i}{P_i^2} - \sum_{i=1}^{3} Z_i \frac{a_i c_i}{P_i^2}$$

$$C_2 = -\sum_{i=1}^{3} X_i \frac{a_i b_i}{P_i^2} + \sum_{i=1}^{3} Y_i \frac{(1-b_i^2)}{P_i^2} - \sum_{i=1}^{3} Z_i \frac{b_i c_i}{P_i^2}$$

$$C_3 = -\sum_{i=1}^{3} X_i \frac{a_i c_i}{P_i^2} - \sum_{i=1}^{3} Y_i \frac{b_i c_i}{P_i^2} + \sum_{i=1}^{3} Z_i \frac{(1-c_i^2)}{P_i^2}$$

$$a_i = -\cos E_i \cos A_i$$

$$b_i = \sin E_i$$

$$c_i = -\cos E_i \sin A_i$$

$$P_i^2 = (x_{j-1} - X_i)^2 + (y_{j-1} - Y_i)^2 + (z_{j-1} - Z_i)^2$$

with i taking the values 1, 2 and 3.

To compute the residual angles, the following equations must be evaluated.

$$r_{A_i} = 3432 \frac{\tan A_i^{m} - \tan A_i^{c}}{1 + \tan A_i^{m} \tan A_i^{c}}$$

$$r_{E_i} = 3432 \frac{\tan E_i^{m} - \tan E_i^{c}}{1 + \tan E_i^{m} \tan E_i^{c}}$$

where

$$\tan A_i^{m} = \frac{c_i}{a_i} \qquad \tan A_i^{c} = \frac{z-Z_i}{x-X_i}$$

$$\tan E_i^{m} = \frac{b_i}{\sqrt{a_i^2+c_i^2}} \qquad \tan E_i = \frac{y-Y_i}{\sqrt{(x-X_i)^2+(z-Z_i)^2}}$$

The improved Model II CPC will compute the coordinates of a rocket and the six residual angles in a single continuous operation requiring approximately two and one-half minutes. This means that a 150 point trajectory can be finished in less than 7 hours, a task that would require approximately 7 man-weeks of hand calculation. These equations must be solved one point at a time in a sequential manner since the co-ordinates of point j in time are used in the computation of the j+1 point. For this reason these equations could not be solved using only the Type 604 multiplier.

General Applications of the CPC.

The two problems outlined above are examples of the type of complex equations that are easily computed on the CPC. The problems mentioned previously, which were worked out on the Type 604, can usually be solved more easily on a CPC and in addition, the CPC can handle many problems that would be out of the question on the 604. Many ordinary differential equations, iterative procedures, and in general, any problem that must be computed in a squential fashion can be done only on the CPC.

The following types of problems are typical examples of the mathematical procedures which can be successfully completed on the Standard CPC or on the Model II CPC.

(1) Solution of simultaneous equations, linear, and non-linear.

(2) Numerical integration and differentiation.

(3) Harmonic analysis

(4) Differencing to any order desired.

(5) Construction of Mathematical Tables.

(6) Interpolation

(7) Summation of series.

(8) Multiplication of series.

(9) Optical ray tracing.

(10) Matrix and determinant calculations.

(11) Method of least squares.

(12) Statistical calculations such as computing variance and covariance.

(13) Curve fitting.

(14) Fourier analysis.

(15) Differential equations.

(16) Certain partial differential equations.

(17) Data reduction.

(18) Generating functions for immediate use in the CPC.

(19) Successive approximation methods.

It can be seen from the problems listed that the CPC is a powerful computational tool which can be used to solve many complex problems.

The computational load of the Computing Branch has been and will continue to be very heavy. The number and complexity of the problems are steadily increasing. More and more scientists are becoming acquainted with the IBM facilities and are thus considering complex problems which could not be undertaken without the aid of a powerful computing installation. To keep abreast of this load, a third CPC has been ordered and is scheduled for delivery in late October, 1951. At the present time the Computing Branch has one of the best equipped IBM facilities in the country and with the delivery of the third CPC this installation will be still further improved.

SUMMARY

This report has covered the various IBM machines which make up a computing facility and the problems which can be solved on these different machines. A comparison of various IBM machines was made which showed the following:

(1). The 604 Calculating Punch was a vast improvement over the mechanical 601 and 602.

(2). The 604 is limited in most cases to a "shotgun" approach which means that it cannot be used to solve problems requiring a sequential solution.

(3). The Standard CPC offered many advantages over the 604. It was the first complete computer brought out by IBM and was able to store numbers, program problems sequentially, and operate in a faster, more flexible way than the 604.

(4). The improved model of the CPC was in turn a definite improvement over the Standard Model and allowed many problems to be solved in a single operation which would have required two or three separate operations on the Standard CPC and many different runs through the 604.

Several problems were illustrated in detail in order to show the type of operations that could be handled on the IBM equipment. These examples showed that certain problems can be readily solved on the 604 while other problems must be solved on the CPC. In general, it can be said that most problems can be solved faster

and more economically on the CPC than on the 604. In addition to the specific problems mentioned, examples were given of the general types of problems that can be solved on the CPC including nineteen different groups of problems that frequently occur in applied mathematics.

This report has attempted to familiarize the reader with the computing equipment available at NOTS and the problems best suited to this type of equipment. With a more intimate knowledge of the equipment and its capabilities, potential users should find it easier to consider solutions to their particular problems which will utilize the speed and flexibility of the present IBM computing facility.

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